Two sterile neutrinos as a consequence
of matter structure[*]

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Abstract

An algebraic argument based on a series of generalized Dirac equations, truncated by an "intrinsic Pauli principle", shows that there should exist two sterile neutrinos as well as three families of leptons and quarks. Then, the influence of these additional neutrinos on neutrino oscillations is studied. As an example, a specific model of effective five–neutrino texture is proposed, where only the nearest neighbours in the sequence of five neutrinos ordered as ν_s, ν'_s, ν_e, ν_μ, ν_τ are coupled through the 5×5 mass matrix. Its diagonal elements are taken as negligible in comparison with its nonzero off–diagonal entries.

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1. Introduction

The existence problem of light sterile neutrinos [1], free of Standard Model gauge charges, is connected phenomenologically with the LSND effect for accelerator ν_μ's [2]. If confirmed, it would avail (jointly with the observed deficits of solar ν_e's [3] and atmospheric ν_μ's [4]) the existence of the third mass–square difference in neutrino oscillations, invoking necessarily at least one kind of light sterile neutrinos, in addition to the familiar three active neutrinos ν_e, ν_μ, ν_τ. From the theoretical viewpoint, however, *light* sterile neutrinos might exist even in the case, when the LSND effect was not confirmed [5]. At any rate, there might be *either* three sorts of light Majorana sterile neutrinos

$$\nu_\alpha^{(s)} = \nu_{\alpha R} + (\nu_{\alpha R})^c \quad (\alpha = e\,,\,\mu\,,\,\tau) \tag{1}$$

being structurally righthanded counterparts of familiar light Majorana active neutrinos

$$\nu_\alpha^{(a)} = \nu_{\alpha L} + (\nu_{\alpha L})^c \quad (\alpha = e\,,\,\mu\,,\,\tau)\,, \tag{2}$$

or some quite new Dirac or Majorana light sterile neutrinos.

The first kind appears in the case of pseudo–Dirac option for neutrinos ν_e, ν_μ, ν_τ [6] that contrasts with the popular seesaw option [7] involving *heavy* sterile neutrinos of the form (1). If the LSND effect did not exist, the seesaw option, operating effectively at low energies with three active neutrinos (2) only, would be phenomenologically most economical, beside the simple option of Dirac neutrinos $\nu_\alpha = \nu_{\alpha L} + \nu_{\alpha R}$. At the same time, the seesaw option would be favourable from the standpoint of GUT idea (say, in the SO(10) version), where a large mass scale of sterile neutrinos of the form (1) could be understood, at least qualitatively. On the other hand, however, such an option would not meet the needs of astrophysics for light sterile neutrinos useful in tentatively explaining heavy–element nucleosynthesis [8].

The sterile neutrinos of the second kind are suggested to exist [9] in the framework of a theoretical scheme based on the series $N = 1, 2, 3, \ldots$ of generalized Dirac equations [10]

$$\left\{ \Gamma^{(N)} \cdot [p - gA(x)] - M^{(N)} \right\} \psi^{(N)}(x) = 0 \,, \tag{3}$$

where for any N the Dirac algebra

$$\left\{ \Gamma_\mu^{(N)} , \Gamma_\nu^{(N)} \right\} = 2g_{\mu\nu} \tag{4}$$

is constructed by means of a Clifford algebra,

$$\Gamma_\mu^{(N)} \equiv \sum_{i=1}^{N} \gamma_{i\mu}^{(N)} \quad , \quad \left\{ \gamma_{i\mu}^{(N)} , \gamma_{j\nu}^{(N)} \right\} = 2\delta_{ij} g_{\mu\nu} \tag{5}$$

with $i , j = 1, 2, \ldots, N$ and $\mu , \nu = 0, 1, 2, 3$. Here, the term $g\Gamma^{(N)} \cdot A(x)$ symbolizes the Standard Model gauge coupling, involving $\Gamma_5^{(N)} \equiv i\Gamma_0^{(N)}\Gamma_1^{(N)}\Gamma_2^{(N)}\Gamma_3^{(N)}$ as well as the color, weak–isospin and hypercharge matrices (this coupling is absent for sterile particles such as sterile neutrinos). The mass $M^{(N)}$ is independent of $\Gamma_\mu^{(N)}$. In general, the mass $M^{(N)}$ should be replaced by a mass matrix of elements $M^{(N,N')}$ which would couple $\psi^{(N)}(x)$ with all appropriate $\psi^{(N')}(x)$, and it might be natural to assume for $N \neq N'$ that $\left[\gamma_{i\mu}^{(N)} , \gamma_{j\nu}^{(N')} \right] = 0$ $i.e.,$ $\left[\Gamma_\mu^{(N)} , \Gamma_\nu^{(N')} \right] = 0$.

The Dirac–type equation (3) for any N implies that

$$\psi^{(N)}(x) = \left(\psi_{\alpha_1 \alpha_2 \ldots \alpha_N}^{(N)}(x) \right) \,, \tag{6}$$

where each $\alpha_i = 1, 2, 3, 4$ is the Dirac bispinor index defined in its chiral representation in which the matrices

$$\gamma_{j5}^{(N)} \equiv i\gamma_{j0}^{(N)}\gamma_{j1}^{(N)}\gamma_{j2}^{(N)}\gamma_{j3}^{(N)} \quad , \quad \sigma_{j3}^{(N)} \equiv \frac{i}{2}\left[\gamma_{j1}^{(N)} , \gamma_{j2}^{(N)} \right] \tag{7}$$

are diagonal (note that all matrices (7), both with equal and different j's, commute simultaneously). The wave function or field $\psi^{(N)}(x)$ for any N carries also the Standard

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Model (composite) label, suppressed in our notation. The mass $M^{(N)}$ gets also such a label. The Standard Model coupling of physical Higgs bosons should be eventually added to Eq. (3) for any N.

For $N = 1$ Eq. (3) is, of course, the usual Dirac equation, for $N = 2$ it is known as the Dirac form [11] of the Kähler equation [12], while for $N \geq 3$ Eqs. (3) give us new Dirac–type equations [10]. All of them describe some spin–halfinteger or spin–integer particles for N odd and N even, respectively. The nature of these particles is the main subject of the present paper (*cf.* also Ref. [10]).

The Dirac–type matrices $\Gamma_\mu^{(N)}$ for any N can be embedded into the new Clifford algebra

$$\left\{ \Gamma_{i\mu}^{(N)} , \Gamma_{j\nu}^{(N)} \right\} = 2\delta_{ij} g_{\mu\nu} \tag{8}$$

[isomorphic with the Clifford algebra introduced for $\gamma_{i\mu}^{(N)}$ in Eq. (5)], if $\Gamma_{i\mu}^{(N)}$ are defined by the properly normalized Jacobi linear combinations of $\gamma_{i\mu}^{(N)}$. In fact, they are given as

$$\Gamma_{1\mu}^{(N)} \equiv \Gamma_\mu^{(N)} \equiv \frac{1}{\sqrt{N}} \sum_{i=1}^N \gamma_{i\mu}^{(N)} \;,\; \Gamma_{i\mu}^{(N)} \equiv \frac{1}{\sqrt{i(i-1)}} \left[\gamma_{1\mu}^{(N)} + \ldots + \gamma_{(i-1)\mu}^{(N)} - (i-1)\gamma_{i\mu}^{(N)} \right] \tag{9}$$

for $i = 1$ and $i = 2, \ldots, N$, respectively. So, $\Gamma_1^{(N)}$ and $\Gamma_2^{(N)}, \ldots, \Gamma_N^{(N)}$ represent respectively the "centre–of–mass" and "relative" Dirac–type matrices. Note that the Dirac–type equation (3) for any N does not involve the "relative" Dirac–type matrices $\Gamma_2^{(N)}, \ldots, \Gamma_N^{(N)}$, solely including the "centre–of–mass" Dirac–type matrix $\Gamma_1^{(N)} \equiv \Gamma^{(N)}$. Since $\Gamma_i^{(N)} = \sum_{j=1}^N O_{ij} \gamma_j^{(N)}$, where the $N \times N$ matrix $O = (O_{ij})$ is orthogonal ($O^T = O^{-1}$), we obtain for the total spin tensor the formula

$$\sum_{i=1}^N \sigma_{i\mu\nu}^{(N)} = \sum_{i=1}^N \Sigma_{i\mu\nu}^{(N)} , \tag{10}$$

where

$$\sigma_{j\mu\nu}^{(N)} \equiv \frac{i}{2} \left[\gamma_{j\mu}^{(N)} , \gamma_{j\nu}^{(N)} \right] \;,\; \Sigma_{j\mu\nu}^{(N)} \equiv \frac{i}{2} \left[\Gamma_{j\mu}^{(N)} , \Gamma_{j\nu}^{(N)} \right] . \tag{11}$$

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Of course, the spin tensor (10) is the generator of Lorentz transformations for $\psi^{(N)}(x)$.

It is convenient for any N to pass from the chiral representations for individual $\gamma_i^{(N)}$'s to the chiral representations for Jacobi $\Gamma_i^{(N)}$'s in which the matrices

$$\Gamma_{j5}^{(N)} \equiv i\Gamma_{j0}^{(N)}\Gamma_{j1}^{(N)}\Gamma_{j2}^{(N)}\Gamma_{j3}^{(N)} \quad , \quad \Sigma_{j3}^{(N)} \equiv \frac{i}{2}\left[\Gamma_{j1}^{(N)},\Gamma_{j2}^{(N)}\right] \tag{12}$$

are diagonal (they all, both with equal and different j's, commute simultaneously). Note that $\Gamma_{15}^{(N)} \equiv \Gamma_5^{(N)}$ is the Dirac–type chiral matrix as it is involved in the Standard Model gauge coupling in the Dirac–type equation (3).

Using the new Jacobi chiral representations, the "centre–of–mass" Dirac-type matrices $\Gamma_{1\mu}^{(N)} \equiv \Gamma_\mu^{(N)}$ and $\Gamma_{15}^{(N)} \equiv \Gamma_5^{(N)}$ can be taken in the reduced forms

$$\Gamma_\mu^{(N)} = \gamma_\mu \otimes \underbrace{\mathbf{1} \otimes \cdots \otimes \mathbf{1}}_{N-1 \text{ times}} \quad , \quad \Gamma_5^{(N)} = \gamma_5 \otimes \underbrace{\mathbf{1} \otimes \cdots \otimes \mathbf{1}}_{N-1 \text{ times}} , \tag{13}$$

where γ_μ, $\gamma_5 \equiv i\gamma_0\gamma_1\gamma_2\gamma_3$ and $\mathbf{1}$ are the usual 4×4 Dirac matrices. For instance, the Jacobi $\Gamma_{i\mu}^{(N)}$'s and $\Gamma_{i5}^{(N)}$'s for $N = 3$ can be chosen as

$$
\begin{aligned}
\Gamma_{1\mu}^{(3)} &= \gamma_\mu \otimes \mathbf{1} \otimes \mathbf{1} & , \quad \Gamma_{15}^{(3)} &= \gamma_5 \otimes \mathbf{1} \otimes \mathbf{1} , \\
\Gamma_{2\mu}^{(3)} &= \gamma_5 \otimes i\gamma_5\gamma_\mu \otimes \mathbf{1} & , \quad \Gamma_{25}^{(3)} &= \mathbf{1} \otimes \gamma_5 \otimes \mathbf{1} , \\
\Gamma_{3\mu}^{(3)} &= \gamma_5 \otimes \gamma_5 \otimes \gamma_\mu & , \quad \Gamma_{35}^{(3)} &= \mathbf{1} \otimes \mathbf{1} \otimes \gamma_5 .
\end{aligned}
\tag{14}
$$

Then, the Dirac–type equation (3) for any N can be rewritten in the reduced form

$$\left\{\gamma \cdot [p - gA(x)] - M^{(N)}\right\}_{\alpha_1\beta_1} \psi_{\beta_1\alpha_2\ldots\alpha_N}^{(N)}(x) = 0 , \tag{15}$$

where α_1 and $\alpha_2 , \ldots , \alpha_N$ are the "centre–of–mass" and "relative" Dirac bispinor indices, respectively (here, $(\gamma \cdot p)_{\alpha_1\beta_1} = \gamma_{\alpha_1\beta_1} \cdot p$ and $\left(M^{(N)}\right)_{\alpha_1\beta_1} = \delta_{\alpha_1\beta_1}M^{(N)}$, but the chiral coupling $g\gamma \cdot A(x)$ involves within $A(x)$ also the matrix γ_5). Note that in the Dirac–type equation (15) for any $N > 1$ the "relative" indices $\alpha_2 , \ldots , \alpha_N$ are free, but still are

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subjects of Lorentz transformations (for α_2 this was known already in the case of Dirac form [11] of Kähler equation [12] corresponding to our $N = 2$).

Since in Eq. (15) the Standard Model gauge fields interact only with the "centre–of–mass" index α_1, this is distinguished from the physically unobserved "relative" indices $\alpha_2, \ldots, \alpha_N$. Thus, it was natural for us to conjecture some time ago that the "relative" bispinor indices $\alpha_2, \ldots, \alpha_N$ are all undistinguishable physical objects obeying Fermi statistics along with the Pauli principle requiring in turn the full antisymmetry of wave function $\psi_{\alpha_1 \alpha_2, \ldots, \alpha_N}(x)$ with respect to $\alpha_2, \ldots, \alpha_N$ [10]. Hence, only five values of N satisfying the condition $N - 1 \leq 4$ are allowed, namely $N = 1, 3, 5$ for N odd and $N = 2, 4$ for N even. Then, from the postulate of relativity and the probabilistic interpretation of $\psi^{(N)}(x)$ we were able to infer that three N odd and two N even correspond to states with total spin $1/2$ and total spin 0, respectively [10].

Thus, the Dirac–type equation (3), jointly with the "intrinsic Pauli principle", if considered on a fundamental level, justifies the existence in Nature of *three and only three* families of spin–$1/2$ fundamental fermions (*i.e.*, leptons and quarks) coupled to the Standard Model gauge bosons. In addition, there should exist *two and only two* families of spin–0 fundamental bosons also coupled to the Standard Model gauge bosons.

For sterile particles, Eq. (15) with any N goes over into the free Dirac–type equation

$$\left(\gamma_{\alpha_1 \beta_1} \cdot p - \delta_{\alpha_1 \beta_1} M^{(N)} \right) \psi^{(N)}_{\beta_1 \alpha_2 \ldots \alpha_N}(x) = 0 \tag{16}$$

(as far as only Standard Model gauge interactions are considered). Here, no Dirac bispinor index α_i is distinguished by the Standard Model gauge coupling which is absent in this case. The "centre–of mass" index α_1 is not distinguished also by its coupling to the particle's four–momentum, since Eq. (16) is physically equivalent to the free Klein–Gordon equation

$$\left(p^2 - M^{(N)\,2} \right) \psi^{(N)}_{\alpha_1 \alpha_2 \ldots \alpha_N}(x) = 0 \ . \tag{17}$$

Thus, in this case the intrinsic Pauli principle requires that $N \leq 4$, leading to $N = 1, 3$ for N odd and $N = 2, 4$ for N even. Similarly as before, they correspond to states with total spin $1/2$ and total spin 0, respectively [9].

Therefore, there should exist *two and only two* spin–$1/2$ sterile fundamental fermions (*i.e.*, two sterile neutrinos ν_s and ν'_s) and, in addition, *two and only two* spin–0 sterile fundamental bosons.

The wave functions or fields of active fermions (leptons and quarks) of three families and sterile neutrinos of two generations can be presented in terms of $\psi^{(N)}_{\alpha_1 \alpha_2 ... \alpha_N}(x)$ as follows

$$
\begin{aligned}
\psi^{(f)}_{\alpha_1}(x) &= \psi^{(1)}_{\alpha_1}(x) \,, \\
\psi^{(f')}_{\alpha_1}(x) &= \frac{1}{4} \left(C^{-1}\gamma_5 \right)_{\alpha_2 \alpha_3} \psi^{(3)}_{\alpha_1 \alpha_2 \alpha_3}(x) = \psi^{(3)}_{\alpha_1 12}(x) = \psi^{(3)}_{\alpha_1 34}(x) \,, \\
\psi^{(f'')}_{\alpha_1}(x) &= \frac{1}{24} \varepsilon_{\alpha_2 \alpha_3 \alpha_4 \alpha_5} \psi^{(5)}_{\alpha_1 \alpha_2 \alpha_3 \alpha_4 \alpha_5}(x) = \psi^{(5)}_{\alpha_1 1234}(x)
\end{aligned}
\tag{18}
$$

and

$$
\begin{aligned}
\psi^{(\nu_s)}_{\alpha_2}(x) &= \psi^{(1)}_{\alpha_2}(x) \,, \\
\psi^{(\nu'_s)}_{\alpha_2}(x) &= \frac{1}{6} \left(C^{-1}\gamma_5 \right)_{\alpha_2 \alpha_3} \varepsilon_{\alpha_3 \alpha_4 \alpha_5 \alpha_6} \psi^{(3)}_{\alpha_4 \alpha_5 \alpha_6}(x) =
\begin{cases}
\psi^{(3)}_{134}(x) & \text{for} \quad \alpha_2 = 1 \\
-\psi^{(3)}_{234}(x) & \text{for} \quad \alpha_2 = 2 \\
\psi^{(3)}_{312}(x) & \text{for} \quad \alpha_2 = 3 \\
-\psi^{(3)}_{412}(x) & \text{for} \quad \alpha_2 = 4
\end{cases} \,,
\end{aligned}
\tag{19}
$$

respectively, where $\psi^{(N)}_{\alpha_1 \alpha_2 ... \alpha_N}(x)$ for active fermions [Eq. (18)] carries also the Standard Model (composite) label, suppressed in our notation, and C denotes the usual 4×4 charge–conjugation matrix. We can see that due to the full antisymmetry in α_i indices for $i \geq 2$ these wave functions or fields appear (up to the sign) with the multiplicities 1, 4, 24 and 1, 6 , respectively. Thus, for active fermions and sterile neutrinos there is given the weighting matrix

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$$\rho^{(a)\,1/2} = \frac{1}{\sqrt{29}} \begin{pmatrix} 1 & 0 & 0 \\ 0 & \sqrt{4} & 0 \\ 0 & 0 & \sqrt{24} \end{pmatrix} \tag{20}$$

and

$$\rho^{(s)\,1/2} = \frac{1}{\sqrt{7}} \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{6} \end{pmatrix} \, , \tag{21}$$

respectively. For all neutrinos (*i.e.*, ν_e, ν_μ, ν_τ and ν_s, ν'_s) described jointly, the overall weighting matrix takes the form

$$\rho^{(a+s)\,1/2} = \frac{1}{\sqrt{36}} \begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & \sqrt{6} & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & \sqrt{4} & 0 \\ 0 & 0 & 0 & 0 & \sqrt{24} \end{pmatrix} \, , \tag{22}$$

if we use the ordering s, s', e, μ, τ. Of course, for all these matrices Tr $\rho = 1$.

Concluding this Introduction, we would like to say that in our approach to families of fundamental particles Dirac bispinor indices ("algebraic partons") play the role of building blocks of composite states identified as fundamental particles. Any fundamental particle, *active* with respect to the Standard Model gauge interactions, contains *one* "active algebraic parton" (coupled to the Standard Model gauge bosons) and *a number $N - 1$* of "sterile algebraic partons" (decoupled from these bosons). Due to the intrinsic Pauli principle obeyed by "sterile algebraic partons", the number N of all "algebraic partons" within a fundamental particle is restricted by the condition $N - 1 \leq 4$, so that only $N = 1, 2, 3, 4, 5$ are allowed. It turns out that states with $N = 1, 3, 5$ carry total spin $1/2$ and are identified with three families of leptons and quarks, while states with $N = 2, 4$ get total spin 0 and so far are not identified. Any fundamental particle, *sterile* with respect to the Standard Model gauge interactions, contains only a number $N \leq 4$ of "sterile algebraic partons", thus only $N = 1, 2, 3, 4$ are allowed. States with $N = 1, 3$ correspond to total spin $1/2$ and have to be identified as two sterile neutrinos, while states with $N = 2, 4$ have total spin 0 and are still to be identified.

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Our algebraic construction may be interpreted *either* as ingeneously algebraic (much like the famous Dirac's algebraic discovery of spin $1/2$) *or* as the summit of an iceberg of really composite states of N spatial partons with spin $1/2$ whose Dirac bispinor indices manifest themselves as our "algebraic partons". In the former algebraic option, we avoid automatically the irksome existence problem of new interactions necessary to bind spatial partons within leptons and quarks of the second and third families. For the latter spatial option see some remarks in the second Ref. [9].

2. A model of five–neutrino texture

In this Section we construct the five–neutrino mass matrix $M = (M_{\alpha\beta})$ under the conjecture that in ordering α, $\beta = s$, s', e, μ, τ of neutrino sequence $\nu_\alpha = \nu_s$, ν'_s, ν_e, ν_μ, ν_τ only the nearest neighbours are coupled through the matrix M. In terms of our presentation of three families of active fermions, Eqs. (18), and two sterile neutrinos, Eqs. (19), such an ordering of five–neutrino sequence tells us that the chain

$$\nu_s \leftrightarrow \nu'_s \leftrightarrow \nu_e \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$$

of neutrino transitions corresponds to the chain

$$\alpha_2 \leftrightarrow \alpha_2\alpha_3\alpha_4 \leftrightarrow \alpha_1 \leftrightarrow \alpha_1\alpha_3\alpha_4 \leftrightarrow \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5$$

of consecutive acts of creation or annihilation of index pairs $\alpha_i\alpha_j$ with $i, j \geq 2$ (pairs of "sterile algebraic partons"), allowed by the intrinsic Pauli principle valid for α_i with $i \geq 2$. In one of these four acts, α_1 must additionally be interchanged with α_2, what should diminish the rate of $\alpha_1 \leftrightarrow \alpha_2\alpha_3\alpha_4$ *versus* $\alpha_1 \leftrightarrow \alpha_1\alpha_3\alpha_4$ (*i.e.*, the magnitude of $M_{s'e}$ *versus* $M_{e\mu}$). One may also argue that the rate of $\alpha_2 \leftrightarrow \alpha_2\alpha_3\alpha_4$ should be still more diminished *versus* $\alpha_1 \leftrightarrow \alpha_1\alpha_3\alpha_4$, (*i.e.*, $M_{ss'}$ *versus* $M_{e\mu}$), as being caused by two such additional interchanges of α_1 with α_2. The allowed alternative chain

$$\alpha_1 \leftrightarrow \alpha_2\alpha_3\alpha_4 \leftrightarrow \alpha_2 \leftrightarrow \alpha_1\alpha_3\alpha_4 \leftrightarrow \alpha_1\alpha_2\alpha_3\alpha_4\alpha_5$$

corresponding to

$$\nu_e \leftrightarrow \nu'_s \leftrightarrow \nu_s \leftrightarrow \nu_\mu \leftrightarrow \nu_\tau$$

does not contain the natural link $\alpha_1 \leftrightarrow \alpha_1\alpha_2\alpha_3$ related to $\nu_e \leftrightarrow \nu_\mu$.

Thus, under the extra assumption that $M_{ss} = 0$, $M_{s's'} = 0$ and $M_{ee} = 0$, we can write

$$M = \begin{pmatrix} 0 & M_{ss'} & 0 & 0 & 0 \\ M_{s's} & 0 & M_{s'e} & 0 & 0 \\ 0 & M_{es'} & 0 & M_{e\mu} & 0 \\ 0 & 0 & M_{\mu e} & M_{\mu\mu} & M_{\mu\tau} \\ 0 & 0 & 0 & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix}, \tag{23}$$

where $M_{\beta\alpha} = M^*_{\alpha\beta}$ due to the hermicity of M. When the CP violation may be ignored in neutrino oscillations, we put $M^*_{\alpha\beta} = M_{\alpha\beta}$ (and $M_{\alpha\beta} > 0$).

Operating with the mass matrix (23), we will make the tentative assumption that, in comparison with its nonzero off–diagonal entries, its nonzero diagonal elements are small enough to be treated as a perturbation of the former. Such a property of M may be related to a tiny neutrino mass scale involved in its diagonal elements. Then, in the zero perturbative order (where $M_{\mu\mu}$, $M_{\tau\tau}$ are put zero), the matrix (23) can be diagonalized exactly, giving the following zero–order neutrino masses $\overset{o}{m}_i$ numerated by $i = 4, 5, 1, 2, 3$:

$$\overset{o}{m}_4 = 0, \ \overset{o}{m}_{5,1} = \mp \left(A - \sqrt{B^2}\right)^{1/2}, \ \overset{o}{m}_{2,3} = \mp \left(A + \sqrt{B^2}\right)^{1/2}, \tag{24}$$

where

$$2A = |M_{e\mu}|^2 + |M_{\mu\tau}|^2 + |M_{ss'}|^2 + |M_{s'e}|^2,$$
$$4B^2 = \left(|M_{e\mu}|^2 + |M_{\mu\tau}|^2 - |M_{ss'}|^2 - |M_{s'e}|^2\right)^2 + 4|M_{\mu\tau}|^2|M_{s'e}|^2. \tag{25}$$

Next, in the first perturbative order with respect to the ratios

$$\xi \equiv M_{\tau\tau}/|M_{e\mu}| \, , \ \chi \equiv M_{\mu\mu}/|M_{e\mu}| \tag{26}$$

we obtain $m_i = \overset{o}{m}_i + \delta m_i$, where

$$\delta m_i = (C_i/D_i)|M_{e\mu}| \tag{27}$$

with

$$\begin{aligned}
C_i &= (\xi + \chi)\overset{o}{m}_i^4 - \left[\xi\left(|M_{e\mu}|^2 + |M_{ss'}|^2 + |M_{s'e}|^2\right) + \chi\left(|M_{ss'}|^2 + |M_{s'e}|^2\right)\right]\overset{o}{m}_i^2 \\
&\quad + \xi|M_{e\mu}|^2|M_{ss'}|^2 \, , \\
D_i &= 5\overset{o}{m}_i^4 - 3\left(|M_{e\mu}|^2 + |M_{\mu\tau}|^2 + |M_{ss'}|^2 + |M_{s'e}|^2\right)\overset{o}{m}_i^2 + |M_{\mu\tau}|^2|M_{s'e}|^2 \, . \tag{28}
\end{aligned}$$

Note that the minus sign possible at m_5 and certain at m_2 is irrelevant since ν_5 and ν_2 are relativistic particles for which only m_5^2 and m_2^2 have physical meaning.

If our argument outlined in the first paragraph of this Section works, the mass matrix elements $M_{ss'}$ and $M_{s'e}$ (which couple the sterile neutrinos ν_s, ν_s' among themselves and ν_s' with the active ν_e, respectively) should be smaller than the elements $M_{e\mu}$ and $M_{\mu\tau}$ (coupling the active neutrinos ν_e, ν_μ, ν_τ), and also the element $M_{ss'}$ should be smaller than $M_{s'e}$: $|M_{ss'}| < |M_{s'e}| < |M_{e\mu}|$. Assuming tentatively $|M_{ss'}| \ll |M_{e\mu}|$ and $|M_{s'e}| \ll |M_{e\mu}|$, it can be seen that in the lowest approximation in the ratios

$$\lambda \equiv |M_{s'e}|/|M_{e\mu}| \, , \ \kappa \equiv |M_{ss'}|/|M_{e\mu}| \tag{29}$$

the formulae (24) and (27) give

$$\overset{o}{m}_4 = 0 \, , \ \overset{o}{m}_{5,1} = \mp\left(\lambda^2 c^2 + \kappa^2\right)^{1/2}|M_{e\mu}| \, , \ \overset{o}{m}_{2,3} = \mp\frac{1}{s}|M_{e\mu}| \tag{30}$$

and

$$\delta m_4 = \xi\frac{\kappa^2 s^2}{\lambda^2 c^2 + \kappa^2}|M_{e\mu}| \, , \ \delta m_{5,1} = \frac{1}{2}\xi\frac{\lambda^2 c^2 s^2}{\lambda^2 c^2 + \kappa^2}|M_{e\mu}| \, , \ \delta m_{2,3} = \frac{1}{2}(\xi c^2 + \chi)|M_{e\mu}| \, , \tag{31}$$

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respectively, where the abbreviations

$$s \equiv \frac{|M_{e\mu}|}{(|M_{e\mu}|^2 + |M_{\mu\tau}|^2)^{1/2}} \; , \; c \equiv \frac{|M_{\mu\tau}|}{(|M_{e\mu}|^2 + |M_{\mu\tau}|^2)^{1/2}} = (1 - s^2)^{1/2} \tag{32}$$

are used. Note that $\sum_i \delta m_i = M_{\mu\mu} + M_{\tau\tau}$, as it should be because of $\sum_i \overset{o}{m}_i = 0$ and $M_{ss} = M_{s's'} = M_{ee} = 0$. For the masses $m_i = \overset{o}{m}_i + \delta m_i$, the formulae (30) and (31) show that $m_5^2 \overset{<}{\sim} m_1^2 \ll m_2^2 \overset{<}{\sim} m_3^2$ and $m_4^2 \ll m_2^2$.

Now, we can calculate the unitary matrix $U = (U_{\alpha i})$ diagonalizing the mass matrix $M = (M_{\alpha\beta})$ given in Eq. (23): $U^\dagger M U = \text{diag}(m_4 \, , m_5 \, , m_1 \, , m_2 \, , m_3)$. In the zero perturbative order with respect to $\xi \, , \chi$ and in the lowest approximation in $\lambda \, , \kappa$, the result is

$$U = \begin{pmatrix} f & -\frac{\kappa}{\lambda c \sqrt{2}} f & \frac{\kappa}{\lambda c \sqrt{2}} f & 0 & 0 \\ 0 & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{\lambda s^2}{\sqrt{2}} & \frac{\lambda s^2}{\sqrt{2}} \\ -\frac{\kappa}{\lambda} f & -\frac{c}{\sqrt{2}} f & \frac{c}{\sqrt{2}} f & -\frac{s}{\sqrt{2}} & \frac{s}{\sqrt{2}} \\ 0 & -\frac{\lambda s^2}{\sqrt{2}} & -\frac{\lambda^2}{\sqrt{2}} & \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{\kappa s}{\lambda c} f & \frac{s}{\sqrt{2}} f & -\frac{s}{\sqrt{2}} f & -\frac{c}{\sqrt{2}} & \frac{c}{\sqrt{2}} \end{pmatrix} , \tag{33}$$

where

$$f = \left(1 + \frac{\kappa^2}{\lambda^2 c^2}\right)^{-1/2} \; , \; \frac{\kappa}{\lambda c} f = \kappa \left(\lambda^2 c^2 + \kappa^2\right)^{-1/2} \tag{34}$$

assuming that $\lambda c \neq 0$. In Eq. (33), a possible small effect of CP violation in neutrino oscillations is ignored by taking $M_{\alpha\beta} = |M_{\alpha\beta}|$.

If not only $M_{ss'} \ll M_{e\mu}$ and $M_{s'e} \ll M_{e\mu}$ but tentatively also $M_{ss'} \ll M_{s'e}$, then beside $\kappa \ll 1$ and $\lambda \ll 1$ also $\kappa \ll \lambda$, and so, we can put $\kappa = 0$ and $f = 1$. As is seen from Eq. (33), in this case the sterile neutrino ν_s is decoupled from $\nu'_s \, , \nu_e \, , \nu_\mu \, , \nu_\tau$ and, therefore, our five–neutrino texture is effectively reduced to a four–neutrino texture, where the masses $m_i = \overset{o}{m}_i + \delta m_i$ given in Eqs. (30) and (31) become

$$m_4 = 0 \, , \; m_{5,1} = \left(\mp \lambda c + \frac{1}{2} \xi s^2\right) M_{e\mu} \, , \; m_{2,3} = \left[\mp \frac{1}{s} + \frac{1}{2} \left(\xi c^2 + \chi\right)\right] M_{e\mu} \, . \tag{35}$$

Here, $m_4^2 \overset{<}{\sim} m_5^2 \overset{<}{\sim} m_1^2 \ll m_2^2 \overset{<}{\sim} m_3^2$.

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When the effect of mixing charged leptons e^-, μ^-, τ^- does not appear or may be ignored in the original lagrangian, then $V = U^\dagger$ is the five–neutrino extension of the lepton counterpart of the familiar Cabibbo–Kobayashi–Maskawa mixing matrix for quarks. In such a situation, the flavor neutrinos ν_α and their states $|\nu_\alpha\rangle$ can be expressed as

$$\nu_\alpha = \sum_i U_{\alpha i} \nu_i \,, \quad |\nu_\alpha\rangle = \nu_\alpha^\dagger |0\rangle = \sum_i U_{\alpha i}^* |\nu_i\rangle \,, \tag{36}$$

where ν_i and $|\nu_i\rangle$ are massive neutrinos and their states, numerated by $i = 4, 5, 1, 2, 3$. Then, in the case of $\kappa^2 \ll \lambda^2 c^2$, for instance, we obtain from Eqs. (33) the following simple mixing of massive neutrinos:

$$
\begin{aligned}
\nu_s &= \nu_4 - \frac{\kappa}{\lambda c}\frac{\nu_5 - \nu_1}{\sqrt{2}} \,, \\
\nu_s' &= \frac{\nu_5 + \nu_1}{\sqrt{2}} + \lambda s^2 \frac{\nu_2 + \nu_3}{\sqrt{2}} \,, \\
\nu_e &= -c\left(\frac{\nu_5 - \nu_1}{\sqrt{2}} + \frac{\kappa}{\lambda c}\nu_4\right) - s\frac{\nu_2 - \nu_3}{\sqrt{2}} \,, \\
\nu_\mu &= \frac{\nu_2 + \nu_3}{\sqrt{2}} - \lambda s^2 \frac{\nu_5 + \nu_1}{\sqrt{2}} \,, \\
\nu_\tau &= s\left(\frac{\nu_5 - \nu_1}{\sqrt{2}} + \frac{\kappa}{\lambda c}\nu_4\right) - c\frac{\nu_2 - \nu_3}{\sqrt{2}} \,.
\end{aligned}
\tag{37}
$$

Here, we have $f = 1$ up to $O(\kappa^2/\lambda^2 c^2)$. Obviously, the assumption $\kappa^2 \ll \lambda^2 c^2$ leading to $\kappa^2 = 0$ is weaker than $\kappa \ll \lambda c$ implying $\kappa = 0$: in the former case the sterile neutrino ν_s is still coupled to ν_s', ν_e, ν_μ, ν_τ, although by a small coefficient $= O(\kappa/\lambda c)$.

3. Five–neutrino oscillations

Finally, in this Section we can evaluate five–neutrino oscillation probabilities making use of the formulae

$$P(\nu_\alpha \to \nu_\beta) = |\langle \nu_\beta | e^{iPL} | \nu_\alpha\rangle|^2 = \delta_{\alpha\beta} - 4\sum_{j>i} U_{\beta j}^* U_{\alpha j} U_{\beta i} U_{\alpha i}^* \sin^2 x_{ji} \,, \tag{38}$$

valid when the CP violation may be ignored (then $U_{\alpha i}^* = U_{\alpha i}$). Here,

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$$x_{ji} = 1.27\frac{\Delta m_{ji}^2 L}{E} \ , \ \ \Delta m_{ji}^2 = m_j^2 - m_i^2 \tag{39}$$

with Δm_{ji}^2, L and E expressed in eV^2, km and GeV, respectively, while $p_i = \sqrt{E^2 - m_i^2} \simeq E - m_i^2/2E$ are eigenvalues of neutrino momentum operator P and L denotes the experimental baseline.

In the case of our mixing matrix (33), valid in the zero perturbative order with respect to ξ, χ and in the lowest approximation in λ, κ, the formulae (38) in the case of $\kappa^2 \ll \lambda^2 c^2$ give

$$\begin{aligned}
P(\nu_e \to \nu_e) &= 1 - c^4 \sin^2 x_{15} - (sc)^2 \left(\sin^2 x_{21} + \sin^2 x_{31} + \sin^2 x_{25} + \sin^2 x_{35} \right) - s^4 \sin^2 x_{32} \\
&\simeq 1 - c^4 \sin^2 x_{15} - (2sc)^2 \sin^2 x_{21} - s^4 \sin^2 x_{32} \ , \\
P(\nu_\mu \to \nu_\mu) &= 1 - \sin^2 x_{32} \ , \\
P(\nu_\mu \to \nu_e) &= s^2 \sin^2 x_{32} - \lambda c s^3 \left(\sin^2 x_{21} - \sin^2 x_{31} - \sin^2 x_{25} + \sin^2 x_{35} \right) \\
&\simeq s^2 \sin^2 x_{32} \ .
\end{aligned} \tag{40}$$

Here, due to Eqs. (38) and (34) we have $\Delta m_{21}^2 \simeq \Delta m_{31}^2 \simeq \Delta m_{25}^2 \simeq \Delta m_{35}^2$ and

$$\Delta m_{15}^2 = 2\xi\lambda s^2 c M_{e\mu}^2 \ , \ \ \Delta m_{32}^2 = 2\frac{\xi c^2 + \chi}{s} M_{e\mu}^2 \ , \ \ \Delta m_{21}^2 = \frac{1}{s^2} M_{e\mu}^2 \ . \tag{41}$$

When $1.27 \Delta m_{32}^2 L_{\text{atm}}/E_{\text{atm}} = O(1)$ for atmospheric ν_μ's and thus $\Delta m_{32}^2 = \Delta m_{\text{atm}}^2 \sim 3.5 \times 10^{-3}$ eV^2 [4], the second formula (40) is able to describe atmospheric neutrino oscillations (dominated in our case by the mode $\nu_\mu \leftrightarrow \nu_\tau$) with maximal amplitude 1. Thus, the second equation (40) leads to the estimation

$$2\frac{\xi c^2 + \chi}{s} M_{e\mu}^2 \sim 3.5 \times 10^{-3} \text{eV}^2 \ . \tag{42}$$

Hence, $\xi + \chi \to 0$ with $c \to 1$ for $M_{e\mu}$ fixed. Also ξ and $\chi \to 0$, since ξ and $\chi \geq 0$.

On the other hand, when $\Delta m_{15}^2 L_{\text{sol}}/E_{\text{sol}} = O(1)$ for solar ν_e's and so, $\Delta m_{15}^2 = \Delta m_{\text{sol}}^2 \sim (6.5 \times 10^{-11}$ or $4.4 \times 10^{-10})$ eV^2 [3] (when considering the "small" or "large" vacuum

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solution), then the first formula (40) has a chance to describe solar neutrino oscillations (dominated now by the mode $\nu_e \to \nu'_s$) with the large amplitude $c^4 = \sin^2 2\theta_{\text{sol}} \sim 0.72$ or 0.90, respectively. In fact, due to $\Delta m^2_{15} \ll \Delta m^2_{32} \ll \Delta m^2_{21}$ the first formula (40) becomes

$$P(\nu_e \to \nu_e) \simeq 1 - c^4 \sin^2 x_{15} - \left(2s^2 c^2 + s^4/2\right) , \tag{43}$$

where the disturbing last term, $2s^2 c^2 + s^4/2 \sim 0.27$ or 0.099, may be too large, but it tends quickly to zero with $c^4 \to 1$. Thus, from the first equation (41) the estimate

$$2\xi\lambda s^2 c M^2_{e\mu} \sim \left(6.5 \times 10^{-11} \text{ or } 4.4 \times 10^{-10}\right) \text{eV}^2 \tag{44}$$

is suggested.

Since $c^2 \sim 0.85$ or 0.95 and $c \sim 0.92$ or 0.97, Eqs. (42) and (44) in the case of $\xi \gg \chi$ (i.e., $M_{\tau\tau} \gg M_{\mu\mu}$) give the estimation

$$\xi M^2_{e\mu} \sim (8.0 \text{ or } 4.2) \times 10^{-4} \text{ eV}^2 , \; \lambda \sim 2.9 \times 10^{-7} \text{ or } 1.1 \times 10^{-5} . \tag{45}$$

Such a tiny value λ shows that $M_{s'e}$ (coupling ν'_s with ν_e) is really very small *versus* $M_{e\mu}$: $M_{s'e} = \lambda M_{e\mu}$. In this case, we get from Eqs. (35)

$$m_{5,1} : m_{2,3} \simeq \lambda sc \mp \frac{1}{2}\xi s^3 \sim \left(1.0 \times 10^{-7} \text{ or } 2.3 \times 10^{-6}\right) \mp 2.4 \times 10^{-6} \left(\text{eV}/M_{e\mu}\right)^2 ,$$
$$m_{2,3} = \mp\frac{1}{s}M_{e\mu} \sim \mp(2.6 \text{ or } 4.4)M_{e\mu} . \tag{46}$$

Thus, in order to obtain, for instance, $|m_2| \simeq m_3 \sim (1 \text{ to } 10)$ eV one should take $M_{e\mu} \sim (0.38 \text{ to } 3.8)$ or $(0.23 \text{ to } 2.3)$ eV. Then, from the first Eq. (44) we infer that

$$\xi \sim 5.6 \times \left(10^{-3} \text{ to } 10^{-5}\right) \text{ or } 7.9 \times \left(10^{-3} \text{ to } 10^{-5}\right) \tag{47}$$

for $c^4 \sim 0.72$ or 0.90, respectively. However, when $M_{e\mu}$ is kept fixed, ξ tends quickly to zero with $c^4 \to 1$.

In the case of Chooz experiment searching for oscillations of reactor $\bar{\nu}_e$'s [13], where it happens that $1.27\Delta m^2_{\text{atm}} L_{\text{Chooz}}/E_{\text{Chooz}} = O(1)$, the first formula (40) with $\Delta m^2_{32} = \Delta m^2_{\text{atm}}$ and $\Delta m^2_{15} = \Delta m^2_{\text{sol}}$ becomes

$$P(\bar{\nu}_e \to \bar{\nu}_e) \simeq 1 - s^4 \sin^2 x_{32} - 2s^2 c^2 \simeq 1 - \left(2s^2 c^2 + s^4\right), \tag{48}$$

since $\Delta m^2_{15} \ll \Delta m^2_{32} \ll \Delta m^2_{21}$. This is consistent with the negative result $P(\bar{\nu}_e \to \bar{\nu}_e) = 1$ of Chooz experiment up to 28% or 10%, but this deviation from 1 tends quickly to zero with $c^4 \to 1$. Note that $U_{e3} = s/\sqrt{2} \sim 0.28$ or 0.16, respectively.

The third formula (40) may imply the existence of $\nu_\mu \to \nu_e$ oscillations with the amplitude equal to $s^2 \sim 0.15$ or 0.05 and the mass–square scale given by $\Delta m^2_{32} = \Delta m^2_{\text{atm}} \sim 3.5 \times 10^{-3} \text{ eV}^2$, while the estimate from LSND experiment [2] is, say, $\sin^2 2\theta_{\text{LSND}} \sim 0.02$ and $\Delta m^2_{\text{LSND}} \sim 0.5 \text{ eV}^2$. Thus, our four–neutrino texture, if fitted to atmospheric and solar results, cannot explain the LSND observation. In order to include the LSND effect, one might depart from our conjecture on nearest–neighbour coupling in the four– or five–neutrino mass matrix.

4. Final remarks: a specific proposal for mass matrix elements

In a specific model of three–neutrino texture discussed by the author previously (*e.g.* [5] and the first Ref. [6]), the following nonzero mass matrix elements were proposed:

$$M_{\mu\mu} = \frac{4 \cdot 80}{9} \frac{\mu}{29}, \; M_{\tau\tau} = \frac{24 \cdot 624}{25} \frac{\mu}{29}, \; M_{e\mu} = \frac{2g}{29}, \; M_{\mu\tau} = \frac{8\sqrt{3}\,g}{29}, \tag{49}$$

where μ and g stood for two small mass scales. Then,

$$M_{\tau\tau} = 16.848\, M_{\mu\mu}, \; M_{\mu\tau} = \sqrt{48}\, M_{e\mu} \tag{50}$$

and from Eqs. (26)

$$\xi = 299.52\, \mu/g, \; \chi = 17.778\, \mu/g = \xi/16.848. \tag{51}$$

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Thus, $\xi \ll 1$ if and only if $g \gg 299.52\mu$, the latter inequality implying $g \gg \mu$ certainly. In the zero perturbative order with respect to ξ or μ/g we put $\mu = 0$.

When accepting the values (49) for $M_{e\mu}$ and $M_{\mu\tau}$, we obtain from Eqs. (32)

$$s = 1/7 = 0.14286 \,, \; c = \sqrt{48}/7 = 0.98974 \,. \tag{52}$$

So, the estimation (42) provided by atmospheric neutrino experiments implies

$$\xi M_{e\mu}^2 \sim 2.4 \times 10^{-4} \text{ eV}^2 \,. \tag{53}$$

Then, due to Eq. (43) and the first Eq. (41), the mass–square difference and oscillation amplitude for solar neutrinos should be

$$\Delta m_{\text{sol}}^2 = 2\xi \lambda s^2 c M_{e\mu} \sim 9.7 \times 10^{-6} \lambda \text{ eV}^2 \,, \; \sin^2 \theta_{\text{sol}} = c^4 = 0.95960 \,, \tag{54}$$

respectively, while the disturbing last term would become smaller than before, giving now $2s^2c^2 + s^4/2 \sim 0.040$.

The values (49) proposed for elements of the three–neutrino mass matrix $M^{(a)} = (M_{\alpha\beta})$ ($\alpha, \beta = e \,, \mu \,, \tau$) can be exactly deduced from the simple ansatz ([10] and the first Ref. [6]):

$$M^{(a)} = \rho^{(a)1/2} \left[\mu(N^2 - N^{-2}) + g(a + a^\dagger) \right] \rho^{(a)1/2} \,. \tag{55}$$

Here,

$$N = 1 + 2a^\dagger a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 5 \end{pmatrix} \tag{56}$$

is the matrix of number of all Dirac bispinor indices α_i (all "algebraic partons") used in Eqs. (18) to present active neutrinos $\nu_e \,, \nu_\mu \,, \nu_\tau$, while

$$a = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix} \,, \; a^\dagger = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & \sqrt{2} & 0 \end{pmatrix} \tag{57}$$

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are (truncated) annihilation and creation matrices of index pairs $\alpha_i\alpha_j$ with $i, j \geq 2$ (pairs of "sterile algebraic partons") included in Eqs. (18) for active neutrinos. The latter matrices satisfy, jointly with the matrix of number of such index pairs,

$$n = a^\dagger a = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{pmatrix} , \qquad (58)$$

the familiar commutation relations

$$[a \, , \, n] = a \, , \ [a^\dagger \, , \, n] = -a^\dagger \, , \qquad (59)$$

and, in addition, the truncation relations $a^3 = 0$ and $a^{\dagger 3} = 0$ consistent with the intrinsic Pauli principle for Dirac bispinor indices α_i with $i \geq 2$ (obviously, neither boson nor fermion canonical commutation relations, $[a \, , \, a^\dagger]_\mp = 1$, are satisfied here). Finally, $\rho^{(a)1/2}$ stands in Eq. (55) for the active–neutrino weighting matrix (20).

In the mass matrix (55), the first term containing μN^2 may be intuitively interpreted as an interaction of all N "algebraic partons" treated on equal footing, while the second involving $-\mu N^{-2}$, as a subtraction term caused by the fact that there is one "active algebraic parton" distinguished (by its external coupling) among all N "algebraic partons" of which $N - 1$, as "sterile" are undistinguishable. This distinguished "algebraic parton" appears, therefore, with the probability $[N!/(N-1)!]^{-1} = N^{-1}$ that, when squared, leads to an additional interaction involving μN^{-2}. The latter interaction should be subtracted from the former in order to obtain for $N = 1$ the matrix element M_{ee} assumed to be zero. The third term in the mass matrix (55) containing $g(a + a^\dagger)$ annihilates and creates pairs of "sterile algebraic partons" and so, is responsible in a natural way for mixing of three active neutrinos.

Of course, the three–neutrino matrix $M^{(a)} = (M_{\alpha\beta})$ $(\alpha, \beta = e \, , \, \mu \, , \, \tau)$ considered in this Section is a submatrix of our five–neutrino mass matrix $M = (M_{\alpha\beta})$ $(\alpha, \beta = s \, , \, s' \, , \, e \, , \, \mu \, , \, \tau)$, *viz.*

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$$
M = \begin{pmatrix} 0 & M_{ss'} & 0 & 0 & 0 \\ M_{s's} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & M_{s'e} & 0 & 0 \\ 0 & M_{es'} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} + \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & M_{e\mu} & 0 \\ 0 & 0 & M_{\mu e} & M_{\mu\mu} & M_{\mu\tau} \\ 0 & 0 & 0 & M_{\tau\mu} & M_{\tau\tau} \end{pmatrix} ,
$$
(60)

where $M_{ss'} = \kappa M_{e\mu}$ and $M_{s'e} = \lambda M_{e\mu}$. Thus, the 2×2 matrix involved in the middle 5×5 matrix plays the role of coupling between the sterile 2×2 matrix $M^{(s)}$ and active 3×3 matrix $M^{(a)}$. If λ were zero, both sterile neutrinos ν_s and ν'_s would be decoupled from the three active. When κ is put zero, ν_s becomes decoupled from ν'_s as well as from ν_e, ν_μ, ν_τ ($M^{(s)}$ is then a zero matrix).

Originally, the ansatz (55) was introduced for mass matrix $M^{(e)} = \left(M^{(e)}_{\alpha\beta} \right)$ ($\alpha, \beta = e, \mu, \tau$) of charged leptons e^-, μ^-, τ^-. In this case, in order to get a small but nonzero value of $M^{(e)}_{ee}$, the quantity $-\mu(1-\varepsilon)N^{-2}$ with a small ε (rather than the quantity $-\mu N^{-2}$) was used in the second term of $M^{(e)}$. Then, the nonzero mass matrix elements were

$$
M^{(e)}_{ee} = \varepsilon \frac{\mu}{29}, \ M^{(e)}_{\mu\mu} = \frac{4(80+\varepsilon)}{25} \frac{\mu}{29}, \ M^{(e)}_{\tau\tau} = \frac{24(624+\varepsilon)}{25} \frac{\mu}{29},
$$
$$
M^{(e)}_{e\mu} = \frac{2g}{29}, \ M^{(e)}_{\mu\tau} = \frac{8\sqrt{3}g}{29} .
$$
(61)

Making the conjecture that for charged leptons diagonal elements of $M^{(e)}$ dominate over its off–diagonal entries (*i.e.*, $\xi \gg 1$ what is certainly true for $g \ll \mu$), we calculated the masses m_e, m_μ, m_τ as eigenvalues of $M^{(e)}$ in the lowest (quadratic) perturbative order with respect to $1/\xi$ or g/μ. Then, we expressed m_τ, μ and ε in terms of m_e, m_μ and $(g/\mu)^2$, obtaining

$$
m_\tau = \left[1776.80 + 10.12112(g/\mu)^2 \right] \ \text{MeV} ,
$$
$$
\mu = \left\{ 85.9924 + O\left[(g/\mu)^2 \right] \right\} \ \text{MeV} ,
$$
$$
\varepsilon = 0.172329 + O\left[(g/\mu)^2 \right] ,
$$
(62)

where the experimental values of M_e and M_μ were taken as the only input. Comparing this prediction for m_τ with the experimental value $m_\tau^{\exp} = 1777.05^{+0.29}_{-0.26}$ MeV [14], we got

$$(g/\mu)^2 = 0.024^{+0.028}_{-0.025} \tag{63}$$

for charged leptons. In such a way, we achieved in the case of charged leptons a really good agreement of our ansatz for $M^{(e)}$ with the experimental mass spectrum (even in the zero perturbative order).

This result has motivated the application of our ansatz for $M^{(e)}$ also to the case of active neutrinos ν_e, ν_μ, ν_τ (corresponding to e^-, μ^-, τ^-). In their case, however, the inverse conjecture $\xi \ll 1$ or $g \gg \mu$ seems natural in view of experimentally suggested large neutrino mixing that is in contradistinction to small mixing of charged leptons [$cf.$ Eq. (63)]. In terms of three active neutrinos alone this ansatz leads to maximal amplitude for atmospheric $\nu_\mu \to \nu_\mu$ oscillations, but it requires introducing at least one sterile neutrino to explain solar $\nu_e \to \nu_e$ oscillations ([5] and the present paper). Even in this case, the LSND effect does not appear, however. Thus, if this effect was confirmed in a clear manner, the conjecture on nearest–neighbour coupling in the four– or five–neutrino mass matrix and, in particular, our ansatz (55) would not be correct for neutrinos.

In this case, a different neutrino texture, also including one or more sterile neutrinos, would be needed. If, on the contrary, the LSND effect was not seen, our four– or five–neutrino texture might be realized in Nature. However, much more economical would be then a three-neutrino texture involving (as $e.g.$ in Ref. [15]) active neutrinos ν_e, ν_μ, ν_τ with the mass hierarchy $m_1^2 \stackrel{<}{\sim} m_2^2 \ll m_3^2$ (in place of $m_1^2 \ll m_2^2 \stackrel{<}{\sim} m_3^2$ valid in the present paper). They ought to be coupled in a different way than in the present paper in order to explain both the atmospheric and solar neutrino results. The argument for our texture would be the absence of LSND effect and, at the same time, the experimental existence of one or two sterile neutrinos (for a possible astrophysical aspect of sterile neutrinos $cf.$ $e.g.$ Ref. [8]).

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